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Urban networks and environmental resilience. Fractal urban aggregates' role

Reti urbane e resilienza ambientale. Il ruolo degli aggregati urbani frattali

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ABSTRACT AND KEYWORDS

Urban networks and environmental resilience
Fractal urban aggregates establish a new paradigm of interpretation, on a natural scale, of the planning process of a territory that evolves between the present and the eco-compatible future. A fractal urban aggregate brings within itself both the morphometric and environmental indicators of minimum sustainable urban development and the laws of direct scaling (self-similar and/or self-affine) for large-area planning, the latter consisting of a finite number of aggregates, according to nature's physical scaling. Fractal analysis in the context of strategic and territorial planning of a large area characterizes urban aggregates in planimetrical terms, as well as the identification of criteria for the development of urban systems that take into account the multiple components. In this sense, the Fractal Dimension is a descriptive criterion, and represents a design parameter.

Keywords: fractal dimension, fractal analysis, fractal urban aggregates, environmental resilience, cities science

Reti urbane e resilienza ambientale
Gli aggregati urbani frattali costituiscono un nuovo paradigma di interpretazione, in scala naturale, del processo di pianificazione di un territorio che evolve tra l'esistente e il futuro eco-compatibile. Un aggregato urbano frattale porta in sè, sia gli indicatori morfometrici e ambientali di sviluppo urbano minimo sostenibile, che le leggi di scala diretta (auto-somigliante e/o autoaffine) per una pianificazione di area vasta, costituita quest'ultima da un numero finito di aggregati, in accordo agli scaling fisici della natura. L'analisi frattale nell'ambito della pianificazione strategica e territoriale di area vasta, caratterizza in termini planimetrici gli aggregati urbani, nonchè la individuazione di criteri per uno sviluppo dei sistemi urbani che tengono conto delle molteplici componenti. In tal senso, la Dimensione frattale è un criterio descrittivo, e rappresenta un parametro progettuale.

Parole chiave: dimensione frattale, analisi frattale, aggregati urbani frattale, resilienza ambientale, scienza delle città

1. Introduction

It can be said that the City is a "complex system", as stated by Cristofaro S. Bertuglia in one of his essays: "*about twenty years ago, it should have been built in such a way as to come, to the conclusion, at the justification of the title itself* ". Today, an exhibition on "the city as a system" "*if it doesn't want to repeat things said over and over and if it doesn't pretend to investigate some epistemological knots that who knows how long they will intrigue philosophers of science, must be constructed assuming the wording as a starting point*" (Bertuglia, 1991). "*So, the city can be conceived as a set of related parts: industrial workplaces, residences, tertiary workplaces, spread over a large chessboard, related to each other through communications and transport*" (Bertuglia, 1991). We will try to give a definition to the concept of "city: as a complex system". There is a large and widespread literature which, at least on an intuitive level, demonstrates that urban and/or territorial systems are complex realities. We assume that the "complexity paradigm" is based on concepts and principles, especially mathematical ones, which have a universal value and are applicable to all non-linear systems. In this context, it is necessary to verify the effective isomorphism between territorial systems and complex systems, given and considering that the Urban Systems respond to the two conditions of non-linearity and openness, necessary and sufficient conditions for any form of self-organization. There are conflicting opinions on this point. (Bertalanffy, 1969). Before delving into this topic, let's give the definitions of a complex system: "*a system can be defined as a complex of interacting elements; interaction means that the elements, P, are connected by relations, R, such that the behaviour of an element p in R is different from what its behaviour would be with respect to another relation R*". For some researchers, Urban Systems are equivalent to complex systems, thanks to the non-linearity of the interactions between urban actors and the plurality of descriptions that such systems can be given. For other scholars, the application of theories of complexity to urban and/or territorial systems is impractical due to the difficulties that arise when trying to uniquely identify such systems, and distinguish them from their environment. It is also necessary to verify that the "city is a complex self-organizing system". Also on this point, there are fundamentally divergent positions between two branches of research: the first states that Urban Systems would be capable, at least in the presence of certain environmental conditions, of acquiring an orderly organization in the absence of top-down planning; complexity theories would make it possible to explain this ability and, possibly, to try to address it; the second trend states that the bifurcations that can be found in the evolution of these systems are over very long time intervals. This concept would make the use of the concept of self-organization in urban research little justified (Bertuglia et al., 2000).

1.1 The city as a complex system

We will analyze these points in detail below, highlighting the different positions: the first point concerns the possibility of studying the city as a complex system; certainly, the interactions between urban actors are of a non-linear type, consisting of different forms ranging from competition, to cooperation, from economies of scale, to the propensity to agglomeration; or even competition for space, for resources and for the conquest of markets (Basili, 1997). Furthermore, the urban phenomenon can be interpreted according to different interpretations: economic, political, social and cultural, or even demographic, spatial or morphological dimensions. This multidimensionality implies that numerous, non-equivalent descriptions are possible for the city (Pumain, 1997). The City, therefore, can be considered a complex system, both on the basis of the non-linearity of the interactions of its elements or, as Casti states, on the basis of the plurality of possible non-equivalent descriptions (Casti, 1986). And, in fact, the city has many of the characteristics that characterize the phenomenology of complex systems: from the exponential increase of political decision-makers, with consequent fragmentation and dispersion of effective authority (Bertuglia, Vaio, 1997) to the structuring according to several levels of organization (Pumain, 1997), from the multiplicity of time-scales "*in the same city numerous and different time scales operate simultaneously, for example, the times for commuting from home to work vary faster than the workplaces and residences, with the consequent phenomena of congestion in the communications network and in housing, as well as buildings generally have a longer duration than the functions for which they were designed) to the apparently acausal behaviour full of surprises*". "*In certain circumstances, it can happen, for example, that a reduction in taxes and interest rates leads to an increase in unemployment, or that neighbourhood regeneration projects through the development of low-cost social housing give rise to neighbourhoods that are worse than those to be rehabilitated, or that the opening of a new road gives rise to an increase in traffic congestion*" (Bertuglia, Vaio, 1997). Another aspect to consider with respect to a systemic conception of the city is linked to the possibility of defining the limits of the city, in order to uniquely identify the urban system. "*By distinguishing between city systems and networks, do we indicate different agglomerations or do we highlight different aspects of the same type of agglomeration or even of the same agglomeration?* " (Bertuglia, Vaio, 1997). The processes of technological innovation and decentralization, as well as the new communication systems, have weakened many of the factors, which previously bound activities to an urban location, and have, with the process of globalization of the economy, increased the freedom of movement of financial capital, goods, information and part of the population. In the past, the problem of the limit was mainly related to the minimum threshold that a city must have in order to be included in a hierarchically ordered taxonomy of settlements; today, however, this problem mainly concerns the higher level, i.e. the identification of the boundaries of the megacity, of the city-region (Tinacci, Masello, 1997). The second point concerns the effective capacity for self-organization of the city, if, on the basis of what has been said above, one accepts, albeit always with due caution, that it can be considered a complex system. According to Prigogine, the city has such capabilities. *"the simplest example of dissipative structures that can be evoked by analogy is the city. A city is different from the countryside around it; the roots of this individualization reside in the relationships it maintains with the adjacent countryside: if these were suppressed, the city would disappear"* (Prigogine,1993). However, it can be admitted that, like dissipative structures, the City satisfies the two indispensable conditions for any form of self-organization: "the non-linearity actions between the elements", and the "openness towards the outside". The city is an open system, which exchanges energy, matter, people (migrations), information and decisions with its environment. This environment is constituted not only by the territory surrounding the city, but also by the system of cities to which it belongs. In other words, the city must always be considered as a system within a system of cities; precisely the exchanges with other cities, even distant ones, are by now predominant compared to those with the surrounding area. The degree of openness of each urban system depends on its situation within the system of cities: larger cities, and above all those specialized in activities of an international nature, are more open to the outside than the others. Among other things, this opening is one of the causes of the fluidity of the city's borders (Pumain,1997). The theory of "complex self-organizing systems" also provides us with a theoretical framework that allows us to integrate two apparently contradictory aspects of cities: their "uniqueness" and their "mutual similarity". Each city is unique, it has an irreducible originality due to the specificity of its location, its history and the territory that surrounds it. At the same time, cities of somewhat comparable size often exhibit similarities in the location of activities, the distribution of growth and even changes in their organization. If we consider the city as a "complex self-organizing system", its evolution can be explained according to a bifurcation diagram with many branches: each branch represents an urban organization that differs qualitatively from the others, but at the base of all the branches there are the same evolutionary processes and mechanisms. In other words, the laws governing the development of the city would be the same for all urban systems, but the existence of bifurcations, of trajectory jumps from one branch to another, would guarantee the presence, in such systems, of distinguishable structures: the fact that, in correspondence with the bifurcations, the choice of one or the other branches is unpredictable, makes the dynamics of each city irreversible and, in a certain sense, also determines its uniqueness (Pumain et al., 1989). This interpretation of the evolution of the city, based on the mutual adaptation of the urban actors in their localization behaviours and on the diffusion of the innovation introduced by the various local actors, should still be considered, according to La Bella, in the state of hypothesis: "*the same the basic idea, ie that the evolution of human settlements, social systems and economies arises from micro-diversity due to errors in the transmission over time of the rules governing specific behaviours and techniques, is absolutely questionable"* (La Bella, 1997).

1.2 Notes on digital innovation processes for the development of urban systems

Going back to digital innovation processes, we can say that they represent an opportunity for urban planning for the development of urban systems, both from the point of view of monitoring infrastructures and the urban environmental state, and of modelling and knowledge of the urban object. Digital technologies also appear useful as tools for citizen involvement and participation. Data driven urbanism, or urban planning based on data collected in a more or less automatic way, opens up various scenarios for more efficient management of the city. This type of urban planning, directed towards better management of the city, finds its application in the monitoring of urban infrastructures. They can be green infrastructures, (as reported by Darte and DeSouza, 2020) that talk about the use of algorithms to analyze googlemaps images, in order to check the distribution and health status of urban greenery. Analysis of this type could also respond more effectively to the demand for public transport that current transport planning techniques cannot satisfy (Macchi, 2006). To this type of analysis, we can add other possibilities for monitoring and data collection of a voluntary and/or participatory nature. The best-known and most widespread case is represented by digital community mapping, or digital mapping of communities. Drawing a map is not a technical operation, but rather the result of choices that are based on the value system of the person drawing (Poli, 2019). Having an active part in the design of a portion of territory and expressing one's values and desires in the urban space through this tool is an important participatory operation, whose possibilities for collection, expression and understanding have been extended to the use of PPGIS (Participatory Public GIS). Mapping or self-mapping can be used in order to understand the extent and scope of the phenomena in which one participates (Belingardi, Pecoriello, 2018). Strictly

speaking of Digital Twin or digital twin, it is a faithful image of a physical process, modelled together with the physical process in question (Batty, 2018). A digital twin is the most accurate and complex reproduction possible of an urban environment, a part of the city or all of it (Castelli et al., 2019). The goal is to collect and systematize as much data as possible on the city in order to have the most complete vision possible of the urban organism at a given time and of its functioning. Since the aim of planning is to change the space and quality of urban life, the digital twin also has the purpose of prefiguring some possible scenarios as a consequence of the choices made and the projects implemented. The idea is placed in the context of urban intelligence, adding the urban component, i.e. *the Senseable city*, to the computerized approach of the smart city. The sensitive city tends to build highly computerized cities, but at the same time, it questions the human side of the city and how to include it, overcoming the predominantly technological approach of the smart city. A digital twin is not a faithful representation of reality, but it must be faithful enough to hold together sufficient data to discuss complex problems (Dembski et al., 2020) by bringing together economic and social processes with the existing environment, and link physical and functional processes to socioeconomic representations (Batty, 2018).
In summary, the research work aims to propose food for thought, in a perspective linked to fractal and network theory, which constitute a paradigm to investigate the complex and multifaceted variety represented by urban systems; a fertile experimental field, on the "City of Tomorrow", with the scientific intent of identifying the different and multiple elements of knowledge, as well as predictive analysis.

2. Materials and methods

2.1 Analysis spatial integration processes

The strength of the reflections of Mela and Preto (Mela and Preto, 1997) lies in the assertion of the systemic and auto-poietic nature of the city. The identification of the processes that lead to the multiple activities that take place in a city allows us to define some criteria according to which it is possible that a "spatial aggregate" can be studied as a self-organized system. (Bertuglia and Staricco, 2000). We will try to define some of these criteria with reference to the spatial integration processes of a socio-economic nature. In fact, every action (economic, political, productive, etc.) of local or urban actors will have its own space-time dimension, with an interaction with other social subjects, with the birth of relationships of interdependence. These are systems of spatial interdependence, i.e. both abstract relational spaces. In fact, urban space is a physical and relational space (Bertuglia and Staricco, 2000). As Mela and Preto state, the different systems develop on different spatial scales, depending on whether the urban actors are co-present or at a distance. These are the so-called integration processes, which are divided as follows: "*first-order integration processes that represent specific systems on a local scale (eg the urban economic system); processes of second-order integration of a horizontal type, which relate the various specific local systems (e.g. the economic system with the political system); second-order integration processes of a vertical type, which relate specific local systems to other specific systems of the same nature, but located elsewhere (supra-local); the top-down and bottom-up approach can be differentiated; firstly, the integration depends on the initiative of the supra-local system (e.g. in which a large multi-lease company, having located one of the activities in an urban centre,* *activates the integration of the economic system of this center into a larger-scale economy ; secondly, on the other hand, the initiative is taken by the local economies themselves; third-order integration processes connect specific supra-local systems of a different nature, effectively generating corporate systems on a regional scale*" (Mela and Preto, 1997). It should be noted that vertical integration can influence the horizontal one, in positive terms, as for example, it has occurred in some cities, such as Amsterdam and Toronto, which have managed to have a position of relevance in the world economic system, involving local operators and decision-makers, implementing careful urban social policies. There are cases in which vertical integration can hinder horizontal integration, as occurs in some metropolises in the southern hemisphere, where a spatial split is evident, due to the contrast between the centre where all the economic and financial activity, and poor and degraded neighbourhoods (Mela, 1996). The analysis of integration processes allows us to estimate to what extent a city satisfies the two conditions of non-linearity and openness, which, as we have said, represent the indispensable boundary conditions for any form of self-organization (Bertuglia and Staricco, 2000). This analysis, therefore, allows us to formulate a judgment on the system of a city and its potential for self-organization. The results of this analysis could come out which indicate systems at the antipodes, for example, a positive opinion can come out, in which systemic properties of a city are deduced, or a negative opinion, equivalent to a third world city, as a mass of unrelated parts or of nodes that refer to separate and uninterconnected networks (Bertuglia and Staricco, 2000). The results could be useful for policymakers, in both cases (Mela and Porto, 1997).

2.2 The pre-industrial city: self-organization

After highlighting the conditions that an urban system must satisfy in order to be able to organize itself, we will consider both the ways in which urban self-organization is implemented and the tools that allow it to be studied. Attention will be focused on two aspects of the city: the morphology (the destruction of the population); the spatial structure (the distribution of activities). The aspect of urban morphology will be evaluated, as the first form of urban self-organization. Some authors have attempted to interpret both the genesis of archaic cities and the development of historic centres that were designed and built without planning tools (Donato and Basili, 1996). According to these authors, "*the unplanned order is the result of a large number of events; that is, it emerges from the sedimentation of a myriad of choices of minimal resistance made in response to a precise and limited set of housing, production, relational needs, etc.* " (Donato and Basili, 1996). In fact, the hypothesis on which this interpretation is based is that urban actors behave according to the "principle of least resistance". In complex systems, non-linearity, with its amplifying effect, underlies counter-intuitive and apparently not random properties and behaviours, which cannot be deduced from an analysis of the constituent elements (Bertuglia and Staricco, 2000). Among these properties, the most interesting is the "capacity for self-organization". Complex systems are able to acquire an orderly and coherent spatial and temporal structure, without the need for an external controller, but on the basis of the pattern of interactions between the constituent elements. The elements each act according to the "principle of least resistance", i.e. trying to obtain the maximum benefit with the minimum effort. According to this principle, each individual belonging to an urban system would aim to satisfy his immediate needs, having no interest in contributing to the formation of an overall order of the urban organism (Bertuglia and Staricco, 2000). Donato and Lucchi Basili affirm that "*the multiplicity of constraints and the complexity of their* *mutual relationships opens the way to an immense spectrum of possible adaptive responses on the part of the urban organism, for the most part totally or partially inadequate; the achievement of an effective organizational solution is thus the result of a long and tiring process of exploration, in which the casual element of discovery and the tension aimed at overcoming the limits of the current situation coexist*" (Donato and Basili, 1996). The process of adaptation, with respect to an overall organization of the urban system that does not respond to social needs, not allowing development, will not be short, indeed in the case of cities, it will be represented by a very long time lapse, both due to the difficulty and the cost of the localization changes. It must also be said that, as Lucchi Basili affirms, "*in the logic of least resistance it is certainly cheaper, in fact, to adapt to safe organizational solutions, because they are already tested and supported by a long cultural tradition; the choices of least resistance, in turn, are fixed in the collective memory of urban civilization, becoming its permanent and therefore reusable heritage*". Urban systems, with respect to the reasoning made, reach a critical condition when there is a social division of labour (Bertuglia and Staricco, 2000). The necessary conditions for a system to self-organize are two: "*non-linearity of the interactions between the constituent elements (complexity of the system) so that small variations of the interactions themselves can amplify until they have consequences at a macroscopic level; the opening of the system, so that it can be subjected to external actions that move it away from equilibrium*". It is evident that two fragments of urban systems will communicate with each other through relationships between the spatial and relational structures. (Donato and Basili, 1996). Without claiming to be exhaustive, the focus is on a "series of unplanned urban fabrics", from the archaic city to the pre-industrial city. The aim is to demonstrate how the self-organization processes have guaranteed the cities, even without the elaboration or application of urban planning tools, orderly and coherent morphologies. The different surrounding conditions, and the continuous processes of consolidation from city to city, have led to a multiplicity of urban forms. (Bertuglia and Staricco, 2000). The proto-urban settlements of the archaic cities of Mesopotamia (3500 BC) expanded uniformly over the plain, with random entrances to the houses and with free spaces, inside the urban agglomeration, devoid of any relational logic. The first real cities were built through the principle of the different functions performed by the different urban actors, in relation to the religious or government role; think of the first buildings (temples, royal palaces, etc.) to which a "public" social significance was attributed. (Bertuglia and Staricco, 2000) "*The main craft, mercantile, etc. activities then begin to concentrate in public spaces, causing a further evolution of the spatial organization of the city and the appearance of new urban functions… thus an elementary logic of definition and maintenance of public urban spaces which leads to the formation of a system of urban relations*". In public spaces, there was a ban on building, both to protect their practicability and because it was considered a usurpation of such spaces. (Frankfort, 1989). Returning to the concept of "least resistance", it can be said that this principle gives the process of spatial structuring a self-propulsive character. (Donato and Basili, 1996).

2.3 The contemporary city: the designed hetero-organization

A historical analysis was carried out with respect to the self-organization processes of the urban morphology, valid up to the pre-industrial era. With the industrial revolution, social, urban, political, etc. innovations are so many, that in many cases they cancel the processes of self-organization. According to the theory of "complex adaptive systems", a system can adapt to its environment only if the latter does not change too rapidly (Bertuglia and Staricco, 2000). It is clear that the industrial city is not able to organize itself adequately. In fact, given that its growth is directly correlated to the innumerable interactions between urban actors, the result is a morphology that is no longer ordered and coherent, but fragmented and congested. A form of cooperation between those who plan from above (public decision-makers) and self-organization from below (local actors) becomes necessary. Cooperation, is commonly defined as: a designed hetero-organization. Designed Hetero-organization is a tool of fractal geometry. A sort of cooperation between "the planner from above" (public decision maker) and "self-organization from below (local actors) becomes necessary. Cooperation, which is commonly defined as: "*designed hetero-organization*".

We will analyse, for example the case of urban morphology, one of the tools suitable for favouring this cooperation, which is represented by fractal geometry, an innovative field of experimentation for cities (Bertuglia and Staricco, 2000). From the theory of dissipative structures, it can be deduced that every "self-organized complex system," which has external perturbations, if they reach a critical value, the system assumes chaotic behaviour. "*the urban organism is, therefore, able to respond to change to the extent that this remains within certain limits; in the presence of a radically changed context, and therefore of completely new needs and problems, which require a radical creative effort and therefore the search for entirely original organizational solutions, the self-organization capacity of the organic city goes into crisis and the danger of a chaotic degeneration becomes concrete*" (Donato and Basili, 1996). In the case of urban systems, the "critical point" was represented by the industrial revolution. In such a context, the various parts of the city begin to organize themselves and adapt autonomously, without an overall coherence; if so, the system is no longer integrated enough to organize itself globally (Bertuglia and Staricco, 2000). The very rapid social transformations, also in terms of economic growth and technological innovation that take place in a city, require particular attention from municipal governments, which find themselves having to deal with investments exceeding their financial resources. The evolution of migratory flows also modifies the ethnic and cultural composition of society. It can be said that the reaction times of the city, in exploring new organizational solutions, are much shorter than the speed of social transformation. Donato and Lucchi Basili argue that, even in such contexts, given that the self-organization processes fail to lead to truly effective solutions, there is a need for top-level planning, in order to make up for the self-organization deficiencies (Donato and Basili, 1997). While, for Western countries, it is a question of ensuring effective integration between the various urban functions, in emerging and/or underdeveloped countries it is necessary to direct the processes of urban growth; the latter becomes increasingly rapid, manifesting itself through the formation of "degraded urban fabrics" (e.g.: favelas) (Bertuglia and Staricco, 2000). Lucchi Basili is convinced that whoever has the task of planning the city must follow the organizational logic of unplanned fabrics, adapting it to new needs; in doing so, the development processes are oriented towards the scales relating to the current transformations. Looking at unplanned adaptation mechanisms, no longer as constraints, but as opportunities, without tolerating phenomena such as illegal use, or other, as a non-adaptation mechanism from below, but rather, a game of interaction between the parties (Lucchi Basili, 1997). The excursus made previously allows us to be able to extend the theory of fractals to complex urban systems. Fractal geometry comes to meet us, helping us to interpret this "hidden non-Euclidean order" related to the "morphological organization of the city" (Bertuglia and Staricco, 2000).

2.4 Fractal urban aggregates and sustainable developments of large areas. Aviable hypothesis

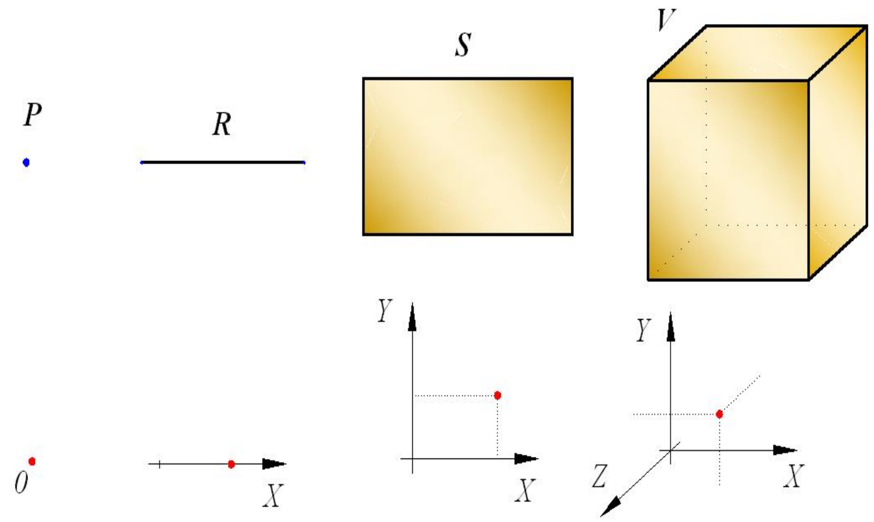
The new urban planning issues focused on environmental sustainability open up new challenges especially with regard to planning support techniques. In fact, the problems that arise from them are numerous and epochal for the approach to the plan, which must be able to analyze the organization of space and consequently outline its sustainability. For instance, the urban layout influences many fundamental processes: air pollution and its serious health consequences; biodiversity linked to the ecological network; the adaptation of the city to climate change, with the relative attenuation of hydraulic risk and heat waves. From this last point of view, numerous studies demonstrate that the city is highly differentiated: it does not have a single climate, but many and much differentiated microclimates (Balena, Leone and Longo, 2020) just as hydraulic danger is differentiated by area (Pelorosso, Gobattoni and Leone, 2018). Since the development of greenery is the most effective solution in both cases, its design cannot be left to chance, which, moreover, often leads to facade operations (green washing). The technologies available today, from GIS to environmental modelling, allow us to better interpret these needs and, therefore, contribute to an innovative "precision urban planning". For this reason, scientific rigor is needed in the definition of space and the fractal logic discussed in this work, wants to give a contribution in this sense, in the belief of the importance of the interdisciplinary approach. The concept of fractal urban aggregate according to Richard Register (Downton, 2009) is that of a "*fraction of the entire city with all the essential components present and arranged for a good interrelation between them and with the natural world and its biology and the resources for human activity*". This definition, largely shareable, certainly falls within the structure (framework) of the entire fractal geometric corpus provided by Benoit Mandelbrot starting from the mid-70s of the last century. These sets are characterized by a non-integer (fractional) dimension, invariant scale, and endowed with some isometric properties and internal similarity typical of Hausdorff measures (Mandelbrot, 1982) defined as self-similarity, or self-affinity, as widely mentioned. These measures, in an urban sense, are essentially characterized by planimetric invariances of urban portions that repeat themselves identically as the observation scale or resolution increases (Downton, 2009). In any case, it is necessary to better define what the scale interval represents with respect to which the fractal urban aggregate is invariant scale. This aspect, of crucial importance for urban planning, falls within the so-called "physical" problem in which fractals "live" in nature and therefore also in the urban and territorial planning context. This remarkable aspect is essentially linked to the concept of scaling, that is, a physical process, or power law, characterized by a scaling exponent, usually a function of the fractal dimension, which represents the parameter or descriptor of said scale interval in terms of invariant measures. The scaling exponent is evaluated, and consequently said interval, according to a least-squares best-fitting criterion and defined within the so-called "physical cut-off limits", beyond which, our measure diverges with respect to the limit resolution, minimum or maximum (De Bartolo et al. 2000). All these definitions are fundamental for a correct fractal characterization of urban developments in the sense of direct scaling and especially for the minimum and maximum identification of representation of the fractal urban aggregate. In this context, and in an exhaustive way, Paul Downton (2009) establishes "physiological" criteria that are specific to the so-called Ecopolis, or fractal eco-sustainable urban city. Among these fractal urban aggregation criteria we recall: the relationships with the processes of the biosphere and the assumptions of human sustainability, the creation of ecological habitats integrated in urban ecosystems, the creation of optimal urban nodes and centers, the creation of connectivity patterns aimed at defining organized urban structures, the creation of networks that have the essential characteristics of large cities. The premises reported so far offer a fairly clear picture of the urban landscape in the scaling of fractal representativeness of the urban development of a territory. The planning criteria to define the organic development processes of urban aggregates must therefore be supported by minimal "physiographic" units connected both to the ecosystems present, or to be developed, and to more or less vast regions, in which the urban ensembles (sets) of aggregation are functional multiples of the larger scale. Such organic developments may have radial forms of self-similar urban expansion, or preferential longitudinal developments in self-affine terms. In this last context, for example, the presence of natural ecological corridors (see Rodriguez-Iturbe and Rinaldo, 1998) can provide a basis for a connective network to be extended to more urban aggregates in line, by superimposing, on said corridors, tree-lined avenues and cycle greenways that are continuously connected to ecological and river parks. We should also remember that Downton himself (2009) indicates the minimum impact and sustainability criteria that must be respected in a fractal urban aggregate. In conclusion, we can affirm that fractal urban aggregates constitute a new paradigm of interpretation, on a natural scale, of the planning process of a territory that evolves between the existing and the eco-compatible future. A fractal urban aggregate carries within itself both the morphometric and environmental indicators of minimum sustainable urban development, and the laws of direct scaling (self-similar and/or self-affine) for a large area planning, the latter consisting of a finite number of aggregates, in accordance with the physical scaling of nature. Fractal analysis in the context of strategic and territorial planning of a large area, characterizes in planimetric terms the urban aggregates, as well as the identification of criteria for the development of "urban systems" that take into account the multiple components. In this sense, the Fractal Dimension is a descriptive criterion, and represents a design parameter.

2.5 The fractal dimension

Fractals are geometric objects obtained through iterative algorithms, that is, a series of instructions repeated according to a predefined sequence. These objects have some characteristics, which can be traced back to two properties:
Self-similarity: "*fractals are invariant under changes in scale; that is, by enlarging a detail of their structure, one finds the exact same starting structure in the case of strictly self-similar fractals (whose algorithm is made up of linear recursive equations), or a structure very similar to the starting one in the case of fractals that are not strictly self-similar (whose algorithm is made up of non-linear recursive equations. In fact, they represent a morphological richness that is never exhausted, at whatever dimensional scale it is observed*"; Fractional dimension: "with their irregular and jagged morphology, fractals occupy space in an intermediate way compared to Euclidean geometric objects; a fractal such as the Koch curve, occupies space with a greater efficiency than a one-dimensional line, but less than a two-dimensional surface; another example is given by the Menger Sponge which is more than a plane, but less than a cube". The fractal dimension contains a lot of information about the geometric properties of the object examined. As a first approximation we can say that it is a measure of the interruption, or irregularity, of a fractal figure when it is observed at a very small scale. The repetitive structure of fractals is measured precisely by the fractal dimension (Devaney, 1990). Over the years, various definitions of the fractal dimension have been formulated, each of which tries to highlight particular characteristics of a certain class of fractal objects,

we remember: the similarity dimension, the Hausdorff dimension, the capacity dimension, the box-counting dimension (or box-counting dimension), the information dimension, the correlation dimension. All definitions that somehow appear to be linked to each other; some make sense only in specific situations and not in others; in some cases the different definitions, as we will see, coincide, in others not all lead to the same numerical result (Rosso, 2005). It is good to dwell on the notion of fractal dimension. The meaning of fractal dimension, being an extension of the concept of dimension normally used to describe ordinary objects, can be easily understood by retracing some concepts of elementary geometry, Euclidean geometry. According to this geometry: an isolated point, or an infinite number of points, constitute a figure of dimension 0; a straight line, as well as a curved line, constitutes a figure of dimension 1; a plane, and any other ordinary surface, constitutes a figure of dimension 2; a cube, as every solid, constitutes a figure of dimension 3 (Mandelbrot, 2003). Along with these primitive concepts, Euclidean geometry assumes that the endpoints of a segment are points, that a finite surface is bounded by lines, and that a finite volume is bounded by surfaces, as in Figure 1.

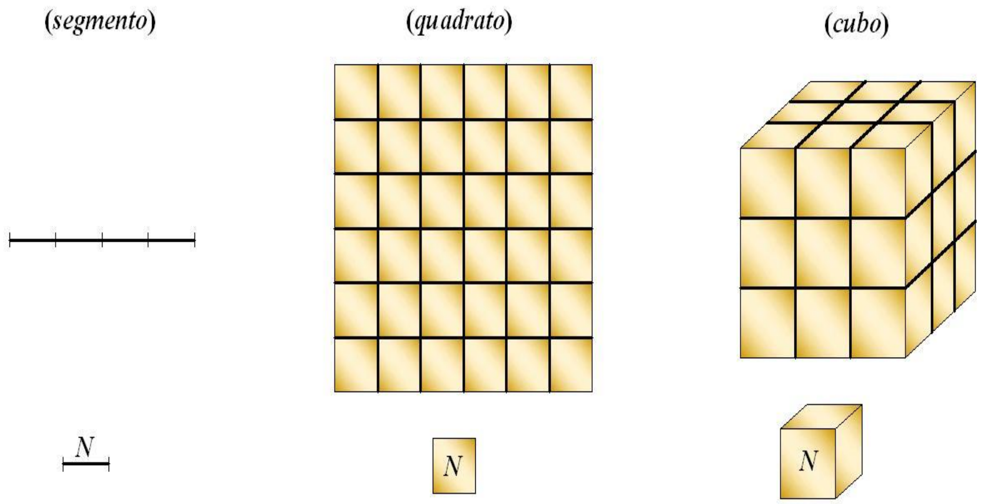
Figure 1. Euclidean geometric entities



This geometric dimension represents, for the above-mentioned figures, the so-called topological dimension DT and is always a natural whole number not greater than three; it is nothing other than a qualitative physical characterization of the dimension implicit in Euclid's geometry, the Euclidean dimension (Mandelbrot, 1987). In fact, Topology studies the qualitative properties of figures, those properties that do not involve the concepts of straight line or plane, but are only functions of the reciprocal positions of the points of the figures, and which consequently do not change by subjecting the figure to arbitrary deformations (Arcidiacono, 2004). Let us now add a trivial observation to what has been said. Considering a purely three-dimensional object, such as a solid cube, we realize that it does not present discontinuity solutions. In other words, the object does not present empty spaces in its structure and uses, so to speak, all the space it has available in the three dimensions. If instead we imagine the structure of a spongy object we notice that, although it occupies a three-dimensional space, it does not occupy it entirely because its structure presents an intricate series of areas not covered by the material of which it is composed. Consequently we can imagine that its dimension is strictly greater than that of a plane, but strictly less than that of a solid, a value between two and three; a fractional value.

It emerges, therefore, that using the concept of dimension, specific to Euclidean geometry, it is impossible to grasp the essence of irregularity. And it is in this regard that Mandelbrot introduces the concept of fractional dimension DF, called fractal dimension which, unlike the Euclidean dimension, can assume a non-integer value, for example it can be a simple fraction (1/2, 5/3, …) or even an irrational number (log4/log3=1.2618…). This measure, in addition to being independent of the chosen scale, is a universal property of the object to which it refers, that is, it is not influenced by the level of relief of the details. The difference between regular and irregular objects is due precisely and exclusively to the dimension, and the most extraordinary thing is that the irregular has found in it a measurable and not simply descriptive indicator (Mandelbrot, 1987).

Figure 2. Subdivision into N parts of a segment, a square and a cube



Let us now reconsider the other peculiar property of fractal objects: self-similarity, or scale invariance, aiming to identify the existence of a mathematical link between this property and the fractal dimension. From an intuitive point of view, self-similarity identifies fractals as mathematical objects formed by parts geometrically similar to the entire figure, but on a reduced scale. We can say that a similar object is composed of a number N of copies of itself rescaled according to a scale factor r, also called reduction factor. So, returning again to the elementary geometric figures (segment, square and cube) it follows, from what has just been stated, that: a segment can be divided into N parts similar to the entire segment, obviously each part of the segment will have a length of 1/N; so a square can be divided into N2 parts similar to the entire square, each of the resulting squares will have an area equal to 1/N2 of the initial square; similarly a cube can be divided into N3 smaller cubes, each having a volume equal to 1/N3 of the initial cube (Arcidiacono, 2004). Since these figures enjoy self-similarity, each small part N that constitutes them has the property of returning identical to the original if enlarged by a scale factor r=N. If for the segment (dim=1) we choose an N=6, which is equivalent to dividing it into four similar parts of length 1/6, it will be necessary to adopt a scale factor r=6 to obtain the same from its N-th part. Thus, if for the square (dim=2) we take an N=2 we obtain four similar

parts of surface 1/4, the scale factor to redefine the same from its N-th part is r=4. Similarly, if we choose for the cube (dim=3) an N=27 we will have twenty-seven similar parts with volume 1/27 and the scale factor to reconstitute it is equal to r=3 (Figure 2).
By performing some calculations, reported in Table 1, it is easy to note that the number of copies N of an object, necessary to constitute it according to a scale factor r, always follows a law that is a function of the size dim of the object itself, or the power law:

$$N = r(N)^{\dim}$$

that is, the number of similar copies N of the object is given by the scale factor r(N) raised to its size *dim*.

Table 1. Dimensional relationship of the geometric elements of the Figure 2

	Dim	***N***	***R***	$N = r^{dim}$
segment	*1*	6	6	$6 = 6^1$
square	*2*	2	4	$4 = 2^2$
cube	*3*	27	3	$27 = 3^3$

With a few simple steps, from $N = r(N)^{\dim}$ we obtain:

$$dim = \frac{\log N}{\log r}$$

where *N* is the number of images obtained from the starting one and r is the scale factor, which determines the order of the homothety and indicates how many times smaller the object obtained is from the original one. And it is precisely this general mathematical relationship that allows the calculation of the fractal dimension (Devaney, 1990). From an experimental point of view, fractals come to our aid in the "sciences of the city". In fact, these objects allow us to reproduce complex and apparently disordered forms, which are governed by a hidden order, such as those generated by self-organization processes (Bertuglia and Staricco, 2000). This infinitely complex morphology is obtained starting from compositional rules that are very simple, thanks to their iteration on multiple dimensional scales. It can be said that fractals follow a principle almost equivalent to that of "minimum resistance" as widely mentioned; in fact, they are characterized by a self-similar morphology, just like the hierarchical structure of the city, based on the logic of the central place. Theoretically, they seem suitable for reproducing those compositional rules that constitute the collective memory, compressing them into an algorithm composed of a few simple instructions, almost like a genetic code, of the "unplanned urban fabrics". Of course, we are still far from stating that, given the relatively recent studies, that the applications of fractal geometry to the study of urban morphology have given satisfactory results, with respect to the construction of the processes of designed hetero-organization. Only future applications will allow us to understand whether the current difficulties can be overcome, or whether they are due to the fact that the morphology of the city does not, in reality, have the characteristics of self-similarity typical of fractals (Bertuglia and Staricco, 2000).
In the field of urban research, two of the most representative applications from a scientific approach point of view are presented: "description of real settlements;

simulation of urban growth processes through algorithms”. As regards the first point, it can be said that if the city is defined as the set of built spaces, it can be considered as a two-dimensional object, that is, a flat area, within which voids of various sizes open up. If we transform the cartographic surveys into digital maps using a scanner, we can verify with the help of computer tools that the surface of these voids, as the survey scale becomes larger, grows according to a constant exponential law: in this case, the exponent represents the fractal dimension of the city. An example of a city with a relative fractal dimension is shown in Figures 3-4 (figures reworked from Google Images). “*We will naturally expect that cities in which the design intervention is more marked and recent will still retain a lower degree of fractality, compared to those in which the aggregation of the fabrics has taken place in a substantial absence of external constraints and over a very long period of time*” (Lucchi Basili, 1996).

Figure 3. An example of a city with relative fractal dimension (figure reworked from Google Images, Tokyo)

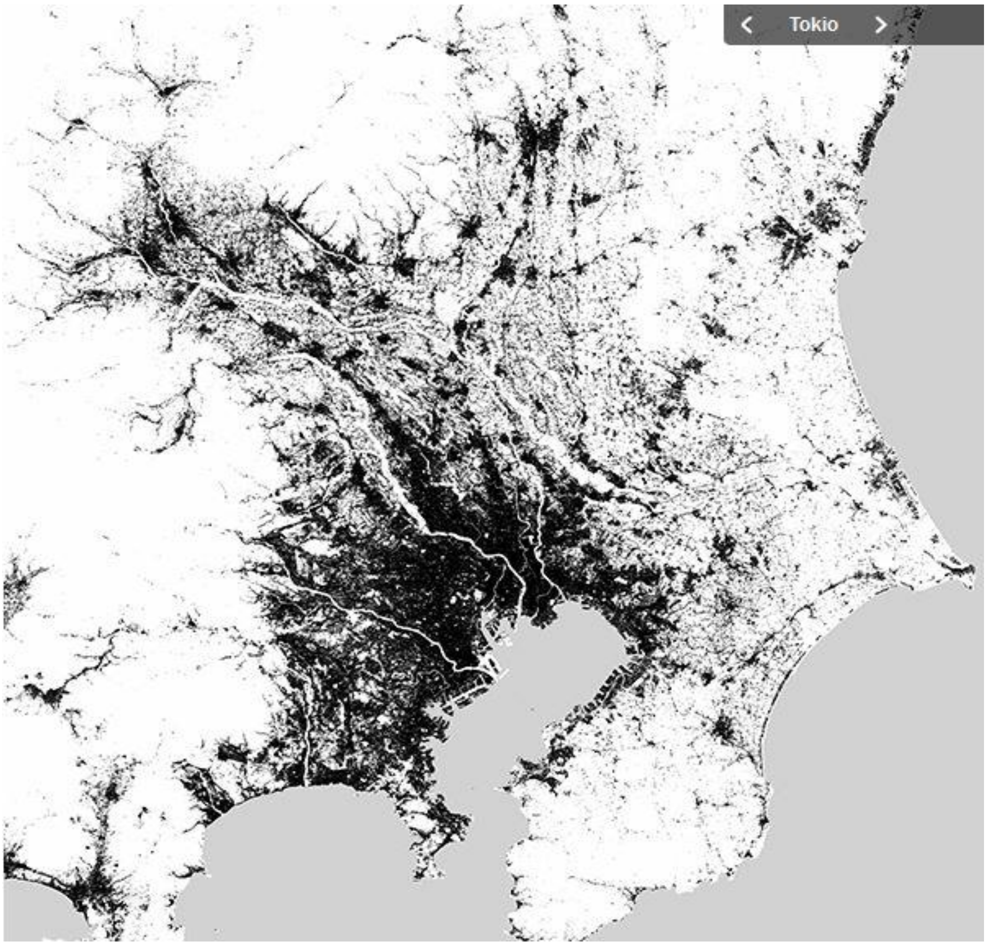
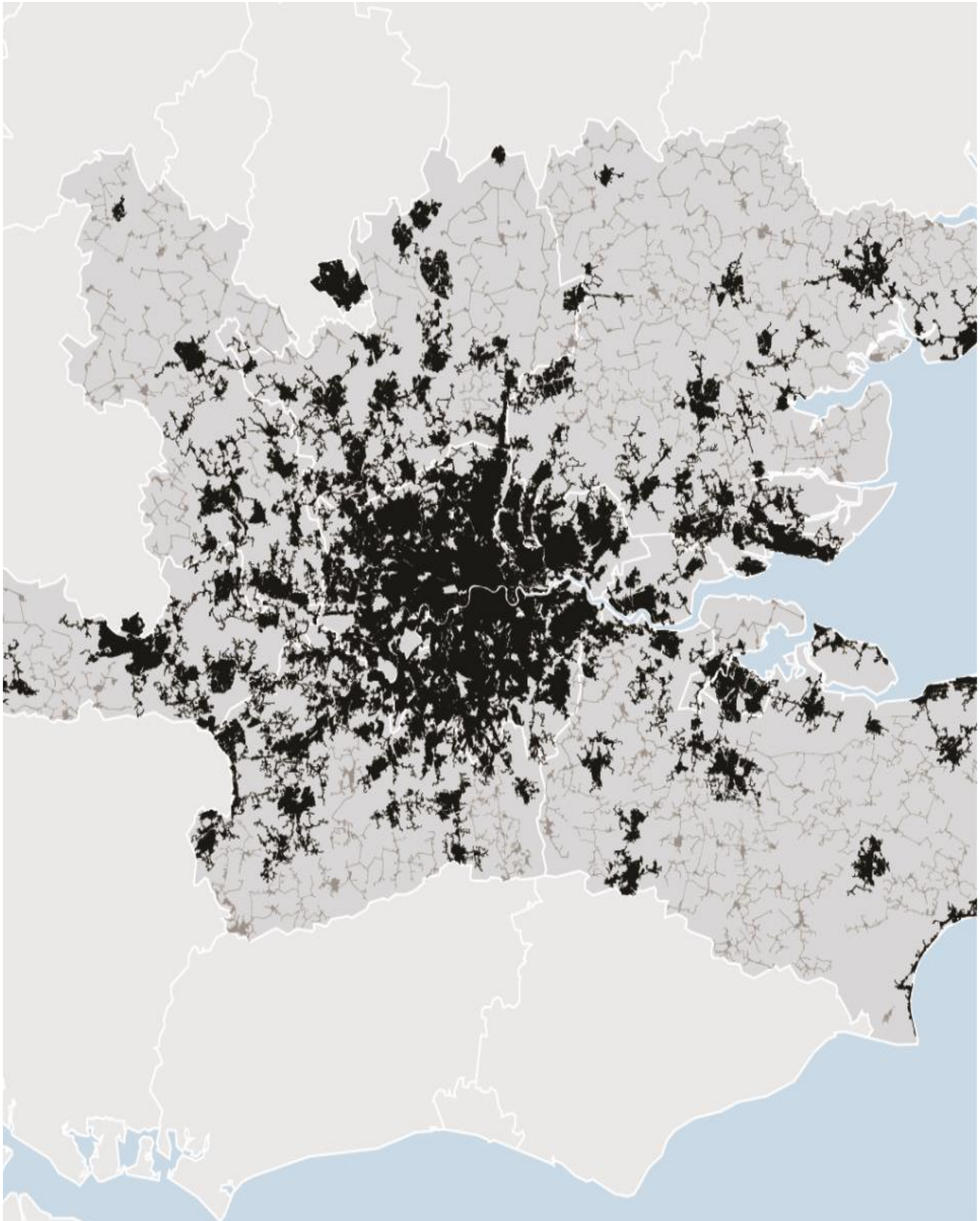


Figure 4. An example of a city with relative fractal dimension (figure reworked from Google Images, London)



3. Results

The “processes of self-organization” concern not only the archaic city but also the historical one, and in particular the medieval one, certainly much less planned from above than the Greek and Roman ones. Indeed, it follows some principles of the Mesopotamian city. The urbanization process is concentrated along the street, creating compactly built fronts and, on the back, pertinent strips intended to serve the new homes; when the matrix path assumes high values, the connections between the extreme poles of the settlement become uncomfortable, due to the excessive distance; the “principle of least resistance” acts, which pushes further construction along the planting paths; connection paths are then created, transversal to those of the plant, as communication between one plant path and another becomes difficult; the progressive saturation of the areas between the routes leads to the formation of quadrangular blocks. It is the result, not of a conscious planning process, but of a self-organization process based on the logic of least resistance (Caniggia and Maffei, 1993). Instead, referring to some examples of network and tree road schemes, the difference in city typology can be seen; we refer to settlements of different urban civilizations, which make evident the social and cultural dimension of the notion of minimum resistance. In fact, the western medieval city and the Islamic one are taken into consideration. In both cases, the criterion against which "resistance" is evaluated is given by mobility. In the European city, this mobility is unconditional, in fact, travelling is of fundamental importance. In the Arab world, on the other hand, mobility is conditioned by the social bonds of family membership. This different relational logic is reflected in the structure of the urban road. In fact, in the western city, the different paths are connected by a network structure, aimed at minimizing the resistance to movement between any two points of the settlement. In the Arab city, the street is a space for relationships. The progressive transformation of a Roman colonial fabric into an Islamic city is evident in medieval Rome, which sees a close interpenetration of private and public space. The case of Rome shows how spontaneous self-organization processes can take over even in historic cities that have been planned according to a practical (rectangular grid) or symbolic (regular polygon) Euclidean rationality model (Bertuglia and Staricco, 2000).

4. Discussion

4.1 The role of spatial self-organization models in urban planning

The "non-linear dynamic models" can be a useful support to better interpret the "designed hetero-organization". In this way we can deal with the urban structure, studying the distribution of the population as well as the activities, in the different areas into which the city is divided. Briefly, we will consider the evolution that urban models have had on a large scale. From the transition from the first generation to the second generation of these models, the role of the latter in a process of designed hetero-organization is shown. Only towards the end of the 1950s the first large-scale urban mathematical models were born. They were based on the use of IT tools, with the simulation of the mobility system or the use of the land inside the city; these models have led to the conception of planning as an applied science. Lowry's model gave rise to many theoretical and experimental developments (Bertuglia et alii, 1987) that were criticized in the mid-1970s. Precisely in those years, we moved from the first generation models to the second generation ones; urban models, from that moment on, are studied in many research centers around the planet (Wegener, 1994). These models are inspired by the study of complex systems (catastrophe theory, bifurcation theory, dissipative structure theory, synergetic process theory). The modeling developed on urban systems can represent an interesting research trajectory, with respect to the quantification of parameters and indicators, useful for urban programming and planning with a view to urban and environmental sustainability. The evolution of hardware supports and the development of software applications, as well as the updating of geographic information systems (GIS), make the outputs of the models more understandable. Recognizing the city as a complex system means recognizing that it must be treated as a non-linear model, i.e. as a system subject to continuous evolution, determined both by internal mechanisms of the system itself and by external forces, albeit to a lesser extent relevant. This innovative conception of urban systems has had a profound impact, both on the characteristics of the models and on the application of the models. For the first point, these models try to identify mechanisms of urban change, which have an exogenous and endogenous origin. These urban mathematical models aim to grasp the endogenous mechanisms of territorial transformation, deriving the urban macro-behaviours from the micro-behaviours of the various actors of the city (Bertuglia, 1991). Therefore, they are dynamic models, in which the relationships between the state variables of the system are not of the algebraic type, but of the differential type and of a qualitative matrix. (Allen et al., 1984). In fact, from the 1970s onwards, there has been an urban growth that has slowed down considerably; that is why, we speak of qualitative transformations, which in any case attract the attention of planners and scholars (Pumain et al., 1989).

Only time will probably tell us who is right and who is wrong, between those who today maintain that our living environments and the settlement models they are inspired by will undergo profound transformations, for example, after the pandemic, so much so as to generate a decline in cities and those who instead believe that they cannot give up cities, the density of relationships, the opportunities, the exchanges that take place in them and that the challenges of the future will be those of countering the unwanted effects of density (congestion, pollution), of correcting the models and practices that have shown inefficiency and little flexibility, of returning to issues such as public health to reduce the gap between people and places. Restarting after the pandemic will not be easy, just as it will not be easy to try to provide answers to the problems highlighted by the crisis. The dense city, elevated to a model of sustainable city for the optimization of transport, for the intensification of social relations, and for economies of scale, according to some scholars has proven incapable of protecting those who live there (Ferrier, 2020), so much so as to reproduce the anti-urban utopia of escape from cities. Other scholars, instead, remind us that cities have survived the wars, revolutions and pandemics of the past because we cannot do without “urbanity” (Wiener & Iton, 2020). Just as they cannot do without “density” (Amphour et al., 2001), “intensity” (De Cunha, 2009) and urban “diversity”, as prerogatives of cities, the political, economic, commercial and cultural centre of human society, in a potential perspective of self-organization. In the face of the crisis, however, the need to reflect on the critical aspects of cities that have also become known, such as: crowding, congestion, social and health inequalities (Shiffers, 2020). As well as on the positive aspects that we had neglected and that we began to appreciate again during the lockdown, such as: the desire for nature, the rejection of the multitude, the sociability of the neighbourhood, the right and safe way of walking or cycling, the possibility of positively influencing air quality with our daily behaviours. All these aspects are closely linked to another way of living, which we have shared, because we have been confined, forced in space and time (Gwiazdzinski, 2020), and which has benefited from the extraordinary acceleration of the integration between the urban phenomenon and the digital world (Balducci, 2020). The difficulty of movement and digital speed have changed the way of conceiving distances and will influence the future of the city, work, mobility, the flows of goods and people. Strengthened by these recent experiences, we must be able to use the crisis and the economic resources earmarked for recovery to implement the transition theorized by the European New Deal (European Commission, 2019). Focusing attention on the spatial dimension of public policies, which has often been lacking in our country (Talia, 2020) and giving priority, with the help of structural and non-emergency measures, to meeting people’s needs and protecting natural and social resources (Raworth, 2017). Far from evoking epochal changes in the organization of the territory and cities, we will have to work to make urban systems more attractive, cooperative and safe, with greater proximity of people to places of production and services. A possible response could be to organize cities according to a network of complementary and hierarchical centralities, articulated in various degrees of compactness, relational intensity, proximity, temporality, proximity to nature and greenery (Da Cunha, 2007).

5. Conclusion

These models tend to describe urban spatial structures, qualitatively different from each other. (Bertuglia and Staricco, 2000). "*Such an approach could then show us that New York, Brussels and Timbuktu correspond to different branches of solutions that the same model presents, for different values of its parameters relating to dimensional and environmental aspects, and for different past events. Only a model of this type, definable as transferable, can be used for predictive purposes in a specific case, while instead a model formulated in terms of a particular problem, and calibrated on it, is purely descriptive, and unable to identify the directions of the change; a model designed to explore the long term must be a transferable model, since the different types of cities we observe are the result of an evolution, and therefore each particular city must be potentially able, under certain conditions, to evolve towards one or the other of these types. It follows that a good model must be a metamodel*" (Allen et al., 1984). With reference to the second point, namely the application of the models, if the city is a complex system, then its dynamics is non-linear; in fact, it is impossible to perfectly define its initial state, accentuated by the presence of fluctuations. (Allen et al., 1984). According to this analytical approach, therefore, it is redundant to use urban models for predictive purposes; if, on the other hand, we refer to the growing speed of change and the growing importance of management tasks, for these reasons urban decision-makers have an even greater need for tools to be able to identify which action to implement or which action to choose between different actions (Bertuglia, 1991). In fact, the appropriate use of mathematical models allows us to develop scenario analyses, as Bertuglia and Vaio affirm, "*models are fundamental for probing, investigating, deepening and, therefore, for defining scenarios, i.e. hypothetical sequences of events based on explicit assumptions and in a particular temporal perspective, which, properly examined, can allow us to put together useful elements on alternative choices and their potential impacts. In this sense, defining scenarios implies, in short, a consistent activity aimed at creating, recording, discussing, synthesizing, presenting and preserving information on future development processes*" (Bertuglia and Vaio, 1997). Unlike predictive models, scenarios include not only quantitative, but also qualitative aspects: based on the analysis of stability and criticalities, the models make it possible to identify possible futures, i.e. the branches in the evolutionary diagram that the system can adopt in absence of planning, or as a result of certain actions on it, for certain critical values of the parameters. The planner in his analyzes or evaluations relating to the elaboration of a "plan", must have the ambition to identify the branches of the evolutionary diagram of the given system, together with its current state and its past history, together with what it could adopt spontaneously, trying to push it, through appropriate actions, towards what appears more favorable. (Bertuglia and Staricco, 2000). This possibility consists "*The role of the urban planner today can be seen as one in which he tries to nudge the system to nudge it towards a more desirable trajectory, rather than trying to force it to advance towards a predetermined and fixed pace . He must decide the best course of action at this precise moment, taking into consideration the long-term context and being ready to continually review his set of decisions*" (Bertuglia et alii, 1992). In an essay, Rabino stated that "*the boundaries between analysis, planning and management,* *that is, become less and less marked, outlining a real process of co-evolution between plan and planned system*" (Rabino, 1997). It is in this perspective that "non-linear dynamic models" can make an important contribution to a process of "hetero-organization design". Among the second generation models, those of spatial self-organization are particularly interesting in this sense. These aspects have been studied and analyzed by some authors: Britton harris and Alan Wilson, as well as by a group of the Department of Physical Chemistry directed by Ilya Prigogine, at the Université Libre de Bruxelles, under the guidance of Peter Allen. It should be emphasized that these models, although highly significant, are only particular cases within a field, that of large-scale urban modeling, which is extremely vast and articulated; a research path, with respect to which there are cities that lend themselves to an easier fractal reading, compared to others.

The analyses reported so far offer a clear picture of the urban landscape in the fractal representativeness scaling of the urban development of a territory. The planning criteria for defining the organic development processes of urban aggregates must therefore be supported by minimal "physiographic" units connected both to the existing ecosystems, or to be developed, and to more or less large regions, in which the urban ensembles (sets) of aggregation are functional multiples of the larger scale. Such organic developments may have radial forms of self-similar urban expansion, or preferential longitudinal developments in self-affine terms. Finally, the city can be understood as a place of valorisation of the collective intelligence of its inhabitants, it calls for a paradigm shift capable of producing a new vision of its mission and its ability to generate an enabling ecosystem, based on the hardware provided by the new quality of spaces and infrastructures and on the software constantly updated by active citizenship, but above all equipped with a new operating system consisting of advanced urban planning and urban policies, capable of responding to the changed demands of contemporaneity.

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Originality
The authors declare that this manuscript is original, has not been published before and is not currently being considered for publication elsewhere, in English or any other language. The manuscript has been read and approved by all named authors and there are no other persons who satisfied the criteria for authorship but are not listed. The authors also declare to have obtained the permission to reproduce in this manuscript any text, illustrations, charts, tables, photographs, or other material from previously published sources (journals, books, websites, etc).

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References

Allen, P.M., Sanglier, M., Engelen, G., Boon, F. (1984). *Evolutionary Spatial Models of Urban and Regional System*, Sistemi Urbani, 6.

Amphoux, P., Grosjean, G., Salomon, J. (2001). *La densitè urbaine, du programme au project urbain* (Rapport de recherche n°142), Lausanne: Institut de recherche sut l'environnement construit, departement architecture.

Arcidiacono, B. (2004). *Spazio, iperspazio, frattali: il magico mondo della geometria frattale*, Di Renzo, Roma.

Balducci, A., (2020). *Covid-19, le città sono la risposta al problema, non il problema*, in urban@it, Centro nazionale di studi per le politiche urbane, in http://www.urbanit.it/covid-19-le-città-sono-la-risposta-al-problema-non-il-problema/.

Batty, M., Longley, P.A. (1994). *Fractal Cities, Academic* Press, London.

Batty, M. (2018). *Digital Twins. Environ. Plan. B.* City Sci, 45, 817-820.

Bertalanffy, Von L, (1971). *General System Theory: Foundations, Developments, Applications*, Braziller, 1969, New York [traduzione italiana: (1971) *Teoria generale dei sistemi. Fondamenti, sviluppo, applicazioni*, ISEDI, Milano].

Bertuglia, C.S., Rabino, G.A., Tadei, R. (1992). *Review of the Main Conceptual Issues Facing Contemporary Urban Planning*, Sistemi urbani, 14, 121-132.

Bertuglia, C.S. (1991). *La città come sistema*, in Bertuglia, C.S., La Bella, A. (a cura di), *I sistemi urbani*, Angeli, Milano 1991; pp. 301-390.

Bertuglia, C.S., Staricco L. (2000). *Complessità, Autoorganizzazione, Città*, Angeli, Milano.

Bertuglia, C.S., Vaio F. (1997). *Introduzione*, in Bertuglia C.S., Vaio F., (a cura di), *La città e le sue scienze*, volume 1, *La città come entità altamente complessa*, Angeli, Milano, xiii-CI.

Caniggia, G., Maffei, G.L. (1993). *Lettura dell'edilizia di base*, Marsilio, Venezia.

Castelli, G., Cesta, A., Diez, M., Ravazzani, P., Rinaldi, G., Savazzi, S., Spagnuolo, M., Strambini, L., Tognola, G., Campana, E. F. (2019). *Urban Intelligence: a Modular, Fully Integrated, and Evolving Model for Cities Digital Twinning*, 2019, IEEE 16th International Conference on Smart Cities: Improving Quality of Life Using ICT IoT and AI (HONET-ICT), Charlotte, NC, USA, pp. 033-037, DOI: 10.1109/HONET.2019.8907962.

Casti, J.L. (1986). *On System Complexity: Identification, Measurement, and Management Approaches*, Springer-Verlag, Berlin, 146-173.

Commissione Europea, 2019, *Il Green New Deal Europeo*, http://eur-lex.europa.eu/legalcontent/IT/TXT/PDF/?uri=CELEX:52019DC0640&from=IT.

Dembski, L., Wossner, U., Letzgus, M., Ruddat, M., Yamu, C. (2020). *Urban Digital Twin for Smart Cities and Citizens: the case study of Herrenberg, Germany*, Sustainability, 12, 2307.

Donato, F., Lucchi Basili, L. (1996). *L'ordine nascosto dell'organizzazione urbana. Un'applicazione della geometria frattale e della teoria dei sistemi auto-organizzati alla dimensione spaziale degli insegnamenti*, Angeli, Milano, 135-136.

Devaney, R.L. (1990). *Caos e frattali, matematica dei sistemi dinamici e applicazioni al calcolatore*, Addison-Wesley, Milano.

Duarte, F., DeSouza, P. (2020). *Data Science and Cities: a critical approach*, Harvard Data Dcience Review.

De Cunha, A., Kaiser, C. (2009). *Densitè, centralitè et qualitè urbaine: la notion d'intensitè, outil pour une gestione adaptative des formes urbaines?*, in Urbia, n° 99, p.13-56.

Da Cunha, A., Matthey, L. (2007). *Le ville et l'urbain: des savoirs èmergents*, Presses polytechniques et universitaires romandes, Lausanne.

Ferrier, J. (2020). *La ville dense a trahi ses habitants*, in Metropolitiques.

Frankfort, H. (1989). *L'urbanistica nell'antica Mesopotania*, Casabella, 559, 53-58.

Gwiazdzinski, L., (2020). *Dèsaturer en jouant sur le temps et sur l'espace*, in Millènaire 3 La prospectivede la metropole de Lyon, https://www.millenaire3.com/texte/-d-auteur/Desaturer-en-jouant-sur-le-temps-et-sur-lespace-par-Luc-Gwiazdzinski.

La Bella, A. (1997). *La modellistica dei sistemi complessi e dei processi di autoregolazione: una breve rassegna critica*, in Bertuglia, C.S., Vaio F. (a cura di), *La città e le sue scienze*, volume I, *La città come entità altamente complessa*, Angeli, Milano, 61-94.

Lucchi Basili, L. (1997). La *geometria frattale dell'organizzazione urbana: oltre la crisi dell'ordine spontaneo*, in Bertuglia, C.S., Vaio F. (a cura di), *La città e le sue scienze*, volume 1, *La città come entità altamente complessa*, Angeli, Milano, 205-238.

Macchi, S. (2006). *Politiche urbane e movimenti di donne: specificità del caso italiano*, in Cortesi, G., Cristaldi, F., Droogleever J., (a cura di) *La città delle donne. Un approccio di genere alla geografia urbana*, Patron, Bologna.

Mandelbrot, B.B. (1987). *Gli oggetti frattali: forma, caso e dimensione*, Einaudi, Torino.

Mandelbrot, B.B. (2005). *Multifractal power law distributions: negative and critical dimension and other "anomalies*", explained bynsimple example, in <<Journal of Statistics Physics>>.

Mela, A., Preto, G. (1997). *Processi autoreferenziali di integrazione spaziale*, in Bertuglia C.S., Vaio F., (a cura di), *La città e le sue scienze*, volume 2, *Le scienze della città*, Angeli, Milano.

Mela A. (1996). *Sociologia delle città*, La Nuova Italia Scientifica, Roma, p.246.

Prigogine, I. (1993). *Le leggi del caos*, Laterza, Roma, p.15.

Poli, D. (2019). *Rappresentare mondi di vita. Radici storiche e prospettive per il progetto di territorio*, Mimesis, Milano.

Pumain, D. (1997). *Ricerca urbana e complessità*, in Bertuglia C.S., Vaio F., (a cura di), *La città e le sue scienze*, volume 2, *Le scienze della città*, Angeli, Milano, 1-45.

Pumain, D., Sanders, L., Saint-Julien, T. (1989). *Villes et auto-organisation*, Economica, Paris.

Raworth, K. (2017). *Doughnut Economics*, Penguin Books, London.

Rabino, G. (1997). Tesi di pianificazione urbanistica, in Bertuglia C.S., Vaio F., (a cura di), *La città e le sue scienze*, volume 3, *La programmazione della città*, Angeli, Milano, 73-94.

Scifferes, S. (2020). *The coronavirus pandemic is already increasing inequality*, The conversation.

Talia, M. (2020). *La ricerca della giusta distanza*, Urbanistica Informazioni, 287-288, INU Edizioni, Roma.

Tinacci Massello, (1997). *Possibilità e limiti dell'autoorganizzazione urbana*, in Bertuglia, C.S., Vaio F., (a cura di), *la città e le sue scienze*, volume 1, *La città, come entità altamente complessa*, Angeli, 1997, Milano, 95-128.

Wegener, M. (1994). *Operational Urban Models. State of the Art*, Journal of the American Planning Associaton.

Wiener, S., Iton, A. (2020). *Le città non sono il problema ma la soluzione*, Internazionale.